Mathematics Challenge 2014

UNIVERSITY OF LINCOLN

For a chance to win an iPad mini or one of our runners-up prizes, submit your typed or neatly written solutions to **maths@lincoln.ac.uk** or by post to **Prof. Andrei Zvelindovsky, School of Mathematics and Physics, University of Lincoln, Lincoln, LN6 7TS**. Please include your full name, postal address and email. The closing date is 15 December, 2014.

1

Estimate the distance from which the tower of Lincoln Cathedral appears the same size as the diameter of the Sun. Assume that the height of the tower is 83 m.

2

Find the right-most digit of the number 7^{2014}

(The 2014-th power of 7).

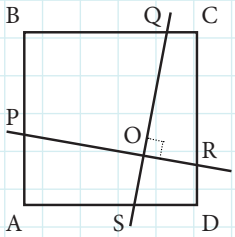
3

Find the right-most digit of the number $7^{(7^{2015})}$

(7 to the power of 2015-th power of7).

4

Given a square ABCD and a point O inside, there are two perpendicular lines through O. They intersect sides AB in P, BC in Q, CD in R, and DA in S. Thus, four quadrangles are formed: APOS, BQOP, CROQ, and DSOR. Prove that the sum of the perimeters of APOS and CROQ is equal to the sum of the perimeters of BQOP and DSOR.



5

How many sequences of length 10 can be composed of two letters A and B (in various proportions) such that no two letters B stand next to each other?

(E.g. ABAABAAAAB is allowed but ABBAAAAAAA is not. You may use binomial coefficients to express your answer).

Notes

Full solutions are required – not just answers – with complete proofs of any assertions you may make.

A winning submission may not necessarily be based on all five problems – so you are encouraged to submit solutions even if you do only some of the problems.